

# Extreme Outages due to Polarization Mode Dispersion: Effects of Optical Filter and “Setting the Clock” Compensation.

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Dependence of the bit-error-rate (BER) caused by amplifier noise in a linear fiber optics line on the fiber birefringence is investigated. We show that the probability distribution function (PDF) of BER obtained by averaging over many realizations of birefringent disorder has an extended tail corresponding to anomalously large values of BER. We specifically discuss dependence of the tail on such details of pulse detection at the fiber output as “setting the clock” and filtering procedures.

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Transmission errors in modern optical telecommunication systems are caused by various impairments (limiting factors). For systems with the rate 40 Gb/s or higher, polarization mode dispersion (PMD) is one of the major impairments. PMD leads to splitting and broadening an initially compact pulse [1–4]. The effect is usually characterized by the so-called PMD vector determining the leading PMD distortion of the pulse [5–7]. It is also recognized that the polarization vector does not provide a complete description of the PMD phenomenon and some proposals aiming to account for “higher-order” PMD effects have been recently discussed [8–11]. Birefringent disorder is frozen (i.e. it does not change at least on the time scales corresponding to the optical signal propagation). There is also a major impairment of another kind, namely the noise originating from amplified spontaneous emission. The amplifier noise is short correlated, i.e. its temporal scale is short compared to the signal width. In this letter we discuss the joint effect of the amplifier noise and the birefringent disorder on the BER. BER is an average over amplifier noise dependent on a current configuration of the birefringent disorder. We focus on describing of such special configurations of the fiber birefringence that produces an anomalously large values of BER, and thus determines reliability of the information transmission.

We propose a theoretical approach to the problem that includes the following steps. Evaluation of the signal BER due to the amplifier noise for a given disorder realization is a preliminary stage of our analysis. The major object of our interest is the PDF of BER (its normalized histogram) where the statistics is collected over different fibers or over the states of a given fiber at different times. We calculate the probability of anomalously large BER. In this letter we analyze the basic case (without compensation) and compare it with the case when the simplest compensation procedure called “setting the clock” is applied. More sophisticated compensation strategies will be discussed elsewhere.

The envelope of the optical field propagating in a given channel in the linear regime (i.e. at relatively low pulse intensity), which is subject to PMD distortion and amplifier noise, satisfies the following equation [12–14]

$$\partial_z \Psi - i \hat{\Delta}(z) \Psi - \hat{m}(z) \partial_t \Psi - i d(z) \partial_t^2 \Psi = \xi(z, t). \quad (1)$$

Here  $z$  is the position along the fiber,  $t$  is the retarded time,  $\xi$  is the amplifier noise and  $d$  is the chromatic dispersion. The envelope  $\Psi$  is a two-component complex field where the two components represent two states of the optical signal polarization. The birefringent disorder is characterized by two random  $2 \times 2$  traceless matrix fields related to the zero-,  $\hat{\Delta}$ , and first-,  $\hat{m}$ , orders in frequency. Birefringence that affects the light polarization is practically frozen ( $t$ -independent) on all the propagation related time scales. The matrix  $\hat{\Delta}$  can be completely excluded from the consideration by the following transformation:  $\Psi \rightarrow \hat{V} \Psi$ ,  $\xi \rightarrow \hat{V} \xi$  and  $\hat{m} \rightarrow \hat{V} \hat{m} \hat{V}^{-1}$ . Here, the unitary matrix  $\hat{V}(z) = T \exp[i \int_0^z dz' \hat{\Delta}(z')]$  is the ordered exponential defined as a formal solution of the equation,  $\partial_z \hat{V} = i \hat{\Delta} \hat{V}$  with  $\hat{V}(0) = \hat{1}$ . Below we always use the renormalized quantities. Then solution of Eq. (1) is  $\Psi = \varphi + \phi$  where,

$$\varphi = \hat{W}(z) \Psi_0(t), \quad \phi = \int_0^z dz' \hat{W}(z) \hat{W}^{-1}(z') \xi(z', t), \quad (2)$$

$$\hat{W}(z) = \exp \left[ i \int_0^z dz' d(z') \partial_t^2 \right] T \exp \left[ \int_0^z dz' \hat{m}(z') \partial_t \right], \quad (3)$$

and  $\Psi_0(t)$  stands for initial pulse.

The additive noise,  $\xi$ , which is an amplification left-over, is zero in average. The statistics of  $\xi$  is Gaussian with spectral properties determined solely by the amplifiers' steady state [15]. The noise correlation time is much shorter than the signal duration, and therefore  $\xi$  can be treated as  $\delta$ -correlated in time. We analyze the case when the pulse propagation distance substantially exceeds the inter-amplifier separation (the system consists of a large number of spans) and consider quantities averaged over

distances larger than the span length  $l_s$ , so that the amplifier noise can be assumed to be short-correlated in space. Summarizing, the Gaussian noise is completely determined by its pair correlation function,

$$\langle \xi_\alpha(z_1, t_1) \xi_\beta^*(z_2, t_2) \rangle = D_\xi \delta_{\alpha\beta} \delta(z_1 - z_2) \delta(t_1 - t_2), \quad (4)$$

where  $D_\xi$  is the noise strength,  $D_\xi = \rho(\omega_0) l_s^{-1}$ , and  $\rho$  is the amplified spontaneous emission (ASE) spectral density generated along a span. This allows to treat discrete and distributed amplification schemes within the same framework. Eqs. (4,2,3) show that  $\phi$  is a zero mean Gaussian field characterized by

$$\langle \phi_\alpha(Z, t_1) \phi_\beta^*(Z, t_2) \rangle = D_\xi Z \delta_{\alpha\beta} \delta(t_1 - t_2), \quad (5)$$

which is statistically independent of both  $d(z)$  and  $\hat{m}(z)$ .

The matrix of birefringence  $\hat{m}$  can be parameterized by a three component real field  $h_j$ , where  $\hat{m} = \sum h_j \hat{\sigma}_j$ , and  $\hat{\sigma}_j$  is the set of three Pauli matrices. The field  $\mathbf{h}$  is zero in average and it is short-correlated in  $z$ . The above transformation  $\hat{m} \rightarrow \hat{V} \hat{m} \hat{V}^{-1}$  guarantees the statistics of  $h_j$  to be isotropic. Since  $\mathbf{h}$  enters the observables described by Eqs. (2,3) in an integral form the central limit theorem (see, e.g., [16]) implies that the field  $h_j$  can be treated as Gaussian field described by the following pair correlation function

$$\langle h_i(z_1) h_j(z_2) \rangle = D_m \delta_{ij} \delta(z_1 - z_2), \quad (6)$$

where the average in Eq. (6) is taken over the birefringent disorder realizations (corresponding to different fibers or to the states of a single fiber taken at different times). If this birefringent disorder is weak the integral  $\mathbf{H} = \int_0^Z dz \mathbf{h}(z)$  coincides with the PMD vector. (Here,  $Z$  is the total length of the fiber.) Thus, and in agreement with [5–7],  $3D_m Z$  measures the mean squared average value of the PMD vector.

We consider the so-called return-to-zero (RZ) modulation format when the pulses are well separated in  $t$ . The signal detection at the line output,  $z = Z$ , corresponds to measuring the pulse intensity,  $I$ ,

$$I = \int dt G(t) |\mathcal{K}\varphi(Z, t) + \mathcal{K}\phi(Z, t)|^2, \quad (7)$$

where  $G(t)$  is a convolution of the electrical (current) filter function with the sampling window function. The linear operator  $\mathcal{K}$  in Eq. (7) stands for an optical filter and a variety of engineering “tricks” applied to the output signal,  $\Psi(Z, t)$ . Ideally,  $I$  takes two distinct values corresponding to the bits “0” and “1”. However, the impairments force deviations of  $I$  from the ideal values. To detect the output signal one introduces some threshold (decision level),  $I_0$ , and declares that the signal codes “1” if  $I > I_0$  and “0” otherwise. Sometimes the information is lost, i.e. an initial “1” is detected as “0” at the output or vice versa. The BER is the probability of

such an event which is measured averaging over many pulses coming through a fiber with a given realization of birefringent disorder,  $\mathbf{h}(z)$ . For successful system performance the BER should be extremely small, i.e. typically both impairments can cause only a small distortion of a pulse. Based on Eq. (7) one concludes that the “0” to “1” change from the input to the output is primarily due to the noise-induced contribution  $\phi$  and, therefore, the probability of such event is insensitive to the birefringence disorder due to Eq. (5). Therefore, anomalously large values of BER are solely due to the “1  $\rightarrow$  0” events. We denote the probability of the “1  $\rightarrow$  0” transition by  $B$ . Because of the smallness of the optical signal-to-noise ratio (OSNR),  $B$  can be estimated using Eqs. (5,7) as the probability of an optimal (saddle-point) fluctuation of  $\phi$  leading to  $I < I_0$ . One derives that the product  $D_\xi Z \ln B$  depends on the disorder, the chromatic dispersion coefficient and the measurement procedure (i.e. the forms of  $\Gamma$  and  $\mathcal{K}$ ), while  $D_\xi Z \ln B$  is insensitive to the noise characteristics.

Out of the variety of detection “tricks” we will discuss here only those correspondent to optical filtering and “setting the clock”. (Other compensation options will be considered elsewhere.) “Setting the clock” procedure is formalized as,  $\mathcal{K}_{cl}\Psi = \Psi(t - t_{cl})$ , where  $t_{cl}$  is the optimal time delay. Since  $D_\xi Z$  is small even weak disorder could produce a large increase in the value of  $B$ . This fact allows a perturbative evaluation of the  $\ln(B/B_0)$  dependence on  $h_j$  (where  $B_0$  is a typical value of  $B$  correspondent to  $h_j = 0$ ). Thus, expanding the ordered exponential (3) in the powers of  $\mathbf{h}$  and retaining only the leading term contributing to the expression for  $B$ , one obtains  $\ln(B/B_0) = \Gamma/(D_\xi Z)$ . If no compensation is applied  $\Gamma = \mu_1 H_3 + O(H^2)$ , and the initial pulse  $\Psi_0$  is assumed to be linearly polarized along  $(1, 0)$ . Note that even for a symmetric initial pulse  $\mu_1 \neq 0$  due to filtering. “Setting the clock” compensation makes  $\mu_1 = 0$  if  $t_{cl}$  is chosen to be exactly equal to  $H_3$ . In this case and also when the output signal is not chirped (this corresponds to the case when there is no chirp in the initial signal and the integral value of chromatic dispersion,  $\int_0^Z dz d(z)$ , is negligible) one gets  $\Gamma = \mu_2(H_1^2 + H_2^2) + O(H^3)$ .

Aiming to demonstrate a qualitative dependence of the parameters  $\Gamma_0 \equiv -D_\xi Z \ln B_0$ ,  $\mu_1$ , and  $\mu_2$  on the measurement procedure, we present here the results of calculations for a simple model case. We rescale both the signal width and its amplitude to unity, thus yielding  $D_\xi Z \ll 1$ ,  $D_m Z \ll 1$ . We assume that the optical filter has the Lorentzian shape:  $\mathcal{K}_f \Psi = \int_0^\infty dt' \exp(-t'/\tau) \Psi(t - t')/\tau$ . Then, as it follows from Eq. (5) the statistics of the inhomogeneous contribution,  $\mathcal{K}\phi$ , is governed by the PDF,  $\mathcal{P}$ :

$$\ln \mathcal{P}(\phi) = -\frac{1}{D_\xi Z} \int dt [\mathcal{K}\phi|^2 + \tau^2 |\partial_t \mathcal{K}\phi|^2]. \quad (8)$$

The inequality  $D_\xi Z \ll 1$  enables one to find  $B$  in the

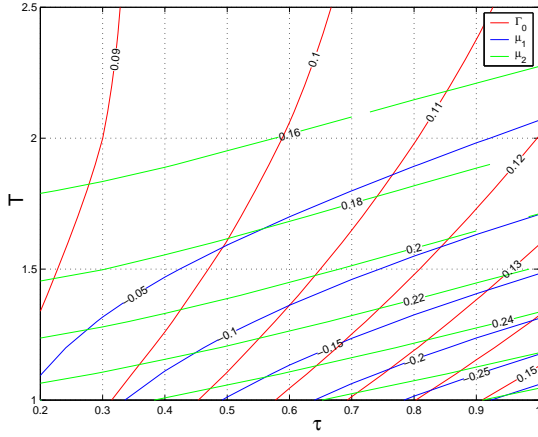


FIG. 1: Dependence of  $\Gamma_0$  and of  $\mu_{1,2}$  on  $T, \tau$  (both measured in the units of the pulse width).

saddle-point approximation. The saddle-point equation is

$$[\tau^2 \partial_t^2 - 1 - uG(t)] \mathcal{K}\phi = uG(t)\mathcal{K}\varphi, \quad (9)$$

where  $u$  is a parameter to be extracted from the self-consistency condition (7). Since  $D_\xi Z$  is a small parameter,  $B$  is estimated by  $\mathcal{P}(\phi_0)$ , where  $\phi_0$  is the solution of Eqs. (7,9) with  $I = I_0$ . Next, we assume that  $G(t) = 1$  at  $|t| < T$  and it is zero otherwise. Then, for a given value of  $u$ , the solution of Eq. (9) can be found explicitly. The value of the parameter  $u$ , however, is not arbitrary, it is fixed inexplicitly by Eq. (7).  $u$ , and thus  $B$  (as functions of  $\tau, T$  and the disorder  $h_j$  through its dependence on  $\mathcal{K}\varphi$ ) can be found perturbatively in  $h_j$ , i.e. as  $u \approx u_0 + \delta u$ ,  $\delta u \ll u_0$ , where  $u_0$  is the solution of the system (7,9) at  $h_j = 0$ . For the Gaussian shape of the initial pulse,  $\Psi_0 = C_g \exp(-t^2/2)$  (where  $C_g$  is enforcing the  $I = 1$  condition for  $\Psi = \Psi_0$ ) and for the  $I_0 = 1/2$  value of the decision level, the numerically found dependence of  $\Gamma_0, \mu_{1,2}$  on  $\tau$  and  $T$  is shown in Fig. 1.

The PDF of  $B$ ,  $\mathcal{S}(B)$  (obtained by averaging over many realizations of the birefringent disorder) can be found by recalculating the statistics of  $H_j$  from Eq. (6) followed by substituting the result into the corresponding expression that relates  $B$  to  $H$  through  $\ln(B/B_0) = \Gamma/(D_\xi Z)$ . Our prime interest is finding the PDF tail correspondent to the values of  $H_j$  essentially exceeding its typical value  $\sqrt{D_m Z}$  which, however, remains to be much smaller than the signal duration. In this range one gets the following estimate for differential probability  $\mathcal{S}(B)dB$ :

$$a) \exp \left[ -\frac{D_\xi^2 Z}{2D_m \mu_1^2} \ln^2 \left( \frac{B}{B_0} \right) \right] \frac{dB}{B}, \quad b) \frac{B_0^\alpha dB}{B^{1+\alpha}}, \quad (10)$$

where (a) marks the basic case, (b) stands for the optimal “setting the clock” case, and  $\alpha \equiv D_\xi/(2\mu_2 D_m)$ . Notice, that the result correspondent to the case (b) shows a steeper decay than in the case (a), which is a natural result of the compensation procedure.

Summarizing, our major result is the emergence of the extremely long tail (10) in the PDF of BER. Note that Eq. (10) shows a complex “interplay” of noise and disorder that may not be deduced from a naive “equal-footing” estimate. Even though an extensive experimental (laboratory and field trial) of our analytical result would be of a great value, some numerics, consistent with Eq. (10) is already available. Thus, Fig. 2a of [17] replotted in log-log variables shows the relation between  $\ln S$  and  $\ln B$  close to linear, which is consistent with Eq. (10b). We are going to analyze more complicated compensation procedures in our subsequent publications.

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